## Experiment 2，液體折射率量測－Abbe 折射率儀 Index of refraction of an unknown liquid－－－Abbe Refractometer

## Principle：

The value n may be written in the form

$$
n=\frac{\sin \frac{1}{2}\left(\delta_{m}+\theta\right)}{\sin \frac{\theta}{2}}
$$

This relation provides us with one or the standard methods of determining the index of refraction of a substance，when it is in the form of a prism．By use of a spectrometer the angles $\theta$ and $\delta_{m}$ may be measured with great accuracy．

## Part 1 ：The prism angle $\theta$

Let the prism be placed on the spectroscope table as shown in Figure 1，the edge of the prism towards the collimator，light falling upon and being reflected from both surfaces．The telescope M is turned until light reflected from face AB enters it，and an image of the slit is seen by reflection in the field of view coinciding with the cross－hair．Now the telescope is turned into position M＇，so that an image of the slit is seen by reflection from AC．
The difference is twice the angle of the prism． This is an illustration of the principle that if a plane reflector is turned through any angle， the reflected ray is turned through twice that angle．


## Part 2 ：The angle of minimum deviation

 $\delta \mathrm{m}$ 。The light ray will be refracted as it passes from air into the prism and will proceed along the line MN．The angle $\delta$ in figure 2 represents the total deviation of the ray resulting from its passage through the prism．From figure 2 we see the following relations：

Figure 1


Figure 2

$$
\begin{aligned}
& \alpha=180^{\circ}-90^{\circ}-\phi_{1}+\theta=90^{\circ}-\phi_{1}+\theta \\
& \beta=90^{\circ}-\phi_{2} \\
& \delta=180^{\circ}-\left(90^{\circ}-\phi_{2}\right)-\left(90^{\circ}-\phi_{1}+\theta\right)=\phi_{1}+\phi_{2}-\theta
\end{aligned}
$$

When $\delta$ is a minimum, its derivative with respect to $\phi_{1}$ is zero. Hence,

$$
\frac{d \delta}{d \phi_{1}}=1+\frac{d \phi_{2}}{d \phi_{1}}=0 \quad \Rightarrow \quad \frac{d \phi_{2}}{d \phi_{1}}=-1 \quad \Rightarrow \quad d \phi_{2}=-d \phi_{1} \quad \Rightarrow \quad \phi_{2}=-\phi_{1}
$$

Therefore angle $\delta$ is a minimum when the beam passed through the prism symmetrically. In your report derive the equation for the index of refraction

$$
\begin{equation*}
n=\frac{\sin \frac{1}{2}\left(\delta_{m}+\theta\right)}{\sin \left(\frac{\theta}{2}\right)} \tag{1}
\end{equation*}
$$

## Procedure:

Since this experiment requires highly accurate work it is necessary to have the spectrometer in complete adjustment.

1. Focus the eyepiece on the cross-hair of the telescope. Direct the telescope toward the light, move the eyepiece tube until the cross-hair are most distinctly observed. Light rays from the cross-hair which enter the eye are than parallel rays.
2. Level the spectrometer. Place a level on the spectrometer plate, adjust the foot screws, turn the level on the plate, adjust again till the spectrometer is absolutely horizontal.
3. Level the spectrometer table. Place a level on the table so that it is parallel to a line through two of the leveling screws, and level by adjusting these screws. Then place the level at right to its first position, and level by adjusting the third screw.
4. Level the telescope and collimator. Place the level on top of the barrel of the telescope, and adjust by use of the leveling screw. In a similar manner adjust the collimator.
5. Adjust slit of the collimator. Rotate the telescope until it is directly opposite the collimator and in alignment with it. Slide the slit assembly in the collimator tube until it is in sharp focus and without parallax with respect to the cross-hair. Remember: Do not touch the eyepiece of the telescope. It has been focused for parallel light.
6. Put the prism on the prism table. The prism table should be leveled such that if the light through the hole in the Gaussian eyepiece, the two reflected images on the cross-hair from the two reflecting faces are both at the same height as the cross-hair.
7. Measure the prism angle $\theta$. Turn the prism table so that light can be reflected from both faces simultaneously. Record the positions at which these reflections are seen in the
telescope. Be able to show that the difference between these two readings in $2 \theta$.
Measure $2 \theta$ three times.
8. Rotate the prism and the telescope to observe the deviated ray. Turn the prism table until minimum deviation is observed and record the readings. Then remove the prism from the table and turn the telescope to observe the straight-through or un-deviated ray. Record the scale-readings for 7 times. The difference between this set of readings and the readings of the light deviation is $\delta$. Decide the minimum value of $\delta$ as $\delta_{m}$.

|  | $1^{\text {st }}$ | $2^{\text {nd }}$ | $3^{\text {rd }}$ | $4^{\text {th }}$ | $5^{\text {th }}$ | $6^{\text {th }}$ | $7^{\text {th }}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $2 \theta\left({ }^{\circ}\right)$ |  |  |  |  |  |  |  |
| $\delta\left(^{\circ}\right)$ |  |  |  |  |  |  |  |

The mean value of $\theta=$ $\qquad$ .
The minimum value of $\delta=\delta_{m}=$ $\qquad$ .

## Problem:

Calculate the index of refraction of the liquid from the formula (1) and determine what liquid you used.

## Apparatus:

Spectrometer, mercury arc, hollow-prism filled with an unknown liquid, level.
Proposed compositions: Water

## Useful hints:

Principle of Abbe refractometer is described as following.

The Abbe refractometer is one of the most accurate and most convenient instruments for determining the index of refraction. The measurement is based on the observation of the refracted beam passing through the substance to be measured and entering a standard prism of high refracting index at grazing incidence.
According to the law of refraction, $\mathrm{n}=\sin (\mathrm{i}) / \sin (\mathrm{r})$, where I and r are the angles of incident and refracted ray n is a constant called the index of refraction. From this equation, it is evident that when light is incident upon a transparent body, there will be a definite angle of refraction corresponding to every angle of incidence. If the refractive index of the body is greater than that of the medium outside, the angles of refraction will be smaller than these corresponding angles of incidence. But the upper limit of the angle of incidence is 90 degrees. There will be a corresponding upper limit for the angle of refraction which determines the largest angle which any ray penetrating the body may make with the normal. This angle is the critical
angle of refraction $r_{c}$. Its value is obtained from the basic equation by substituting $\mathrm{i}=90^{\circ}$ or $\sin (\mathrm{i})=1$ where $\mathrm{n}=1 / \sin \left(r_{c}\right)$

Consequently a measurement of the critical angle of refraction gives immediate information concerning the index of refraction.

To an observer at 0 , inside a medium of refractive index $n$, all rays incident upon the surface from above are confined, upon refraction, within a cone corresponding to the angles of refraction from zero up to the critical angle, as illustrated in fig. 1, no illumination exists from any direction whose angle with the normal is greater than this critical angle. Consequently the critical ray makes a boundary between a dark and a light area. This is more clearly illustrated in fig. 2. For monochromatic illumination this boundary will be a sharp line. For measurements of the critical angle as commonly made in the laboratory, the circle of fig. 2, illustrates the appearance of the field of view of the observing telescope.
Since it is not possible as a rule, to make a direct measurement of $r_{c}$, equation (1) can not be

figure 1.

directly
applied to the
determination of indices of refraction. Moreover, since this method is commonly used to measure the index of refraction of a liquid by placing it in contact with a prism, it is of advantage to derive an expressing to take the place of equation (1) upon the assumption that the incident beam, upon passing into the prism, passes from a medium of index $n_{x}$ into the glass of index $n_{g}$. Under these circumstances, for the grazing incident ray, $\mathrm{i}=90^{\circ}$ (ray c in
fig. 2) we have $1=\left(n_{g} / n_{x}\right) \sin (\mathrm{r})$ and for the ray emerging from the glass into the air (ray c).
In addition $r+r \prime=\Lambda$, where $\Lambda$ is the refracting angle of the prism. By elimination of $r$ and $r$ ' from these equations we finally obtain

$$
n_{x}=\sin \Lambda \sqrt{n_{g}^{2}-\sin ^{2} i^{\prime}}-\cos \Lambda \sin i^{\prime} .
$$

This equation expresses the index of refraction n of the medium in contact with the prism for
the condition of grazing incidence (ray c) of the light. For greater convenience of measurement the refractometer built on this principle are equipped with a calibrated scale. The Abbe refractometer measures refractive indices of solids and liquids over the range of 1.3 to 1.7 and is provided with a practically linear scale which can be read to 0.001 and is accurate to 0.0002 . The essential feather of this instrument lies in the used of a pair of so called Abbe prism, two similar 45 -degrees= prisms of high refractive index. A drop of the liquid whose index is desired is placed upon the hypotenuse of one prism, and the hypotenuse of the other prism is placed upon that of the first, so that the two together from a cubical block of glass. It is then evident that for slightly convergent or divergent light incident from an appropriate direction upon the junction of the two prism, a pair will be transmitted and a part totally reflected if the index of the liquid is less than that of the prisms. A telescope focused for parallel light as shown in fig. 3 will show the characteristic divided field. The index of the liquid is then read directly from the calibrated scale. The calibration is made for sodium light, but the instrument can also be used with daylight illumination. This is made possible by the introduction of a compensator consisting of two direct vision prism. They can be rotated in opposite directions at equal rates, and form together a system of variable dispersion. In this way, an amount of dispersion can be introduced to counteract that of the specimen under examination. The compensator carries a scale by means of which a measurement of the dispersion of the material under examination may be obtained. The term "dispersion" in this case means the difference in diffracting index for the wavelength of 6563 angstrom (red), and 4861 angstrom (blue), respectively. Tables are supplied from which this difference may be obtained from the compensator reading.


In the measurements of the refractive index, light is directed into the prisms and along the axis of the telescope by means of a mirror attached to the instrument. The eyepiece is focused simultaneously on the cross-hair and the scale by properly turning the eyepiece.
Using sodium light, the adjustment for an index of refraction reading consists in turning the lower adjusting screw in such a way that a double field half bright half dark with a sharp dividing edge passing through the center of the cross-hair is seen.

Then the index of refraction is read off the scale to the third decimal and the fourth decimal is conveniently estimated. When white light is used, the compensator must be adjusted before final adjustment of the prisms as described below in the procedure.

## Procedure:

General directions: The student is cautioned to handle the refractometer with extreme care.

The prisms are made of flint glass; they are brittle and at the same time extremely soft. Therefore they are easily broken and scratched. Be sure to keep the surfaces of the prisms clean and to use only clean liquid on them.

1. The scale and the cross-hair in the circle are focused by turning the eyepiece.
2. The illumination of the scale is adjusted by turning the small tube projecting at the lower end of the instrument.
3. To lift the upper prism off for applying the sample to be tested, turn the small knob attached to the prism into such a position, that the arrow mark at the side points upward; if it points sideward, the prism is locked in position. With the knob in the free position, swing the upper prism out into its rest position.
4. By means of a smooth glass rod apply a drop of the liquid to be tested of the fixed prism and bring the upper prism back on top of the lower one.
5. Start a sodium burner and adjust its position, open the reflecting mirror of the upper prism and if necessary, adjust it for good illumination of the circle seen through the telescope; if necessary adjust the tube providing the illumination of the scale.
6. Adjust the positions of the prisms by turning the knob at the lower right until in the middle of the bright circle a clear boundary line is observed between a dark gray sector in the lower half and a bright sector in the upper half of the circle. Make a fine adjustment of the prism position by turning the knob at the lower right until the boundary line passes through the center of the cross-hair.
7. Read off the index of refraction on the illuminated scale directly up to the third decimal and estimate the value of fourth.
8. Read and record the temperature of the thermometer attached to the instrument.

Repeat this procedure with other samples. Repeat one of the measurements using white light. This requires the operation of the built-in compensator. The compensator is adjusted by turning the knob at the middle of the instrument. Start with the compensator at the zero mark. Adjust the prisms gradually until a bright red-orange light appears across the middle of the circle. This will be the adjustment close to the final position. For further refinement turn the compensator knob until the color disappears and a sharp boundary line is obtained separation a gray sector in the lower part and a bright sector in the upper part of the circle opening. Read just the prisms until the boundary coincides with the center of the cross-hair. Read the value of the index of refraction and the thermometer as before and compare with the result of the previous measurement.


